Section 3.3 Logarithmic Functions and their Graphs

**Disclaimer: ONLINE HW is easier than the notes BUT QUIZ & TEST covers NOTES, CLASSWORK AND HW. REVIEW ALL**

Without a calculator, approximate the solution of the following equations

a) $2^x = 10$ b) $3^x = 10$

For $x > 0, a > o, a \neq 1,$

$y = \log_a x$ if and only if $x = a^y$

The function $f(x) = \log_a x$ is called a **logarithmic function with base *a*.**

1) Rewrite each of the following equations in logarithmic form (if possible). If not possible, say why.

a. $4^x = 64$ b. $5^x = \frac{1}{125}$ c. $2^x = -32$

2) Use the definition of a logarithmic function to evaluate each logarithm at the indicated value of x.
No calculator

a. $f(x) = \log_4 x, x = 16$ b. $f(x) = \log_2 x, x = 64$

c. $f(x) = \log_3 x, x = \frac{1}{81}$ d. $f(x) = \log_5 x, x = 1$

3) Use a calculator to evaluate the function given by $f(x) = \log x$

a. x=100 b. $x = \frac{1}{5}$ c. x=3.25 d. x= -4

4) Simplify No calculator

a. $\log_5 1$

b. $\log_{\sqrt{11}} \sqrt{11}$

c. $8^{\log_8 30}$

d. $\log \sqrt[5]{10}$

e. $\log_5 \sqrt[3]{5^7}$

f. $\log_{81} 9$

g. $\log \frac{1}{1000}$

5) Solve No calculator

a. $\log_5 y = \log_5 16$

b. $\log(4-3x) = \log(x+2)$

c. $\log_3(x^2+4) = \log_3 29$

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To sketch the graph of $y = \log_a x$, use the fact that the graphs of inverse functions are reflections of each other in the line y=x ( the x and y variables are interchanged). $y = \log_a x$ and $y = a^x$ are inverses of each other.

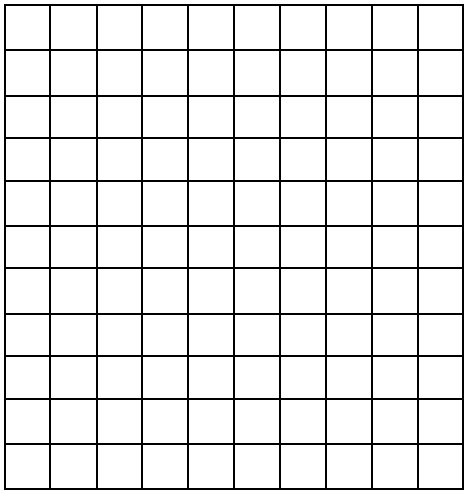
6) On the same coordinate plane, sketch the graph of each function

a) $f(x) = 4^x$ b) $g(x) = \log_4 x$

- Construct a table of values for f(x) No calculator

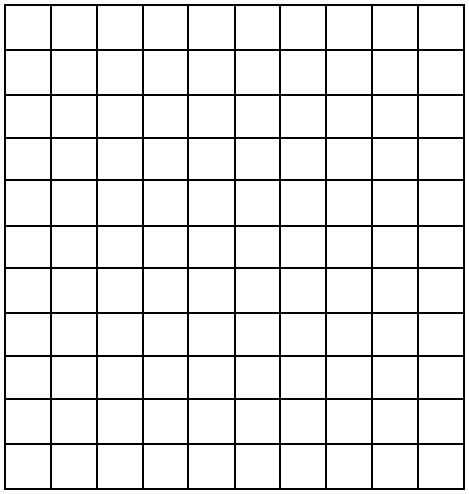
| x | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| f(x) | | | | | | |

- Since $g(x) = \log_4 x$ is the inverse of $f(x) = 4^x$ the graph of $g(x) = \log_4 x$ is obtained by interchanging the x and y variables of $f(x) = 4^x$. In other words, plot the points $(f(x), x)$



7) Sketch the graph of $f(x) = \log\left(\frac{x}{4}\right)$. Identify the domain, x-intercept, and vertical asymptote.

| x | 0 | 1 | 3 | 4 | 5 | 7 |
|---|---|---|---|---|---|---|
| f(x) | | | | | | |



8) Describe the graph as a transformation of the graph of $f(x) = \log_3 x$
State the domain and vertical asymptote.

a) $g(x) = \log_3 x - 3$

b) $h(x) = \log_3 (x + 2)$

Section 3.3 Logarithmic Functions and their Graphs

**The Natural Logarithmic Function**

The function defined by $f(x) = \log_e x = \ln x, x > 0$ is called the natural logarithmic function.

Inverses: $f(x) = e^x$ and $g(x) = \ln x$

Use a calculator to evaluate the function given by $f(x) = \ln x + 1$ for each value of x to four decimal places.

a) x=73.25 b) x=0.4 c) x=-2 d) x= $2 + \sqrt{3}$

**Properties of Natural Logarithms**

1. $\ln 1 = 0$ because $e^0 = 1$
2. $\ln e = 1$ because $e^1 = e$
3. $\ln e^x = x$ and $e^{\ln x} = x$ (inverse property)
4. If $\ln x = \ln y$ then x=y (one to one property)

9) Use the properties of natural logarithms to simplify each expression.

a) $\ln e^{1/2}$ b) $e^{\ln 8}$ c) $15 \ln 1$ d) $\frac{\ln e}{6}$ e) $\ln \sqrt[9]{e^5}$

10) Find the domain of each function.

a) $f(x) = \ln(x + 5)$ b) $f(x) = \ln(3 - x)$ c) $f(x) = \ln x^3$

11) Students in a mathematics class were given an exam and then retested monthly with an equivalent exam. The average scores for the class are given by the human memory model $f(t) = 78 - 17 \log(t + 1), 0 \le t \le 12,$ where t is time in months.

a) What was the average score on the original exam (t=0)?
b) What was the average score after 3 months?
c) What was the average score after 11 months?